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FIRST PASSAGE TIMES FOR COMBINATIONS OF RANDOM LOADS

BY

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FIRST PASSAGE TIMES FOR COMBINATIONS OF RANDOM LOADS

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0. ABSTRACT

Structures are subject to changing loads from various sources. In many instances these loads fluctuate in time in an apparently random fashion. Models are considered for which the stress put on the structure by various loads simultaneously can be described by a regenerative process. The distribution of the first time until the stress on the structure exceeds a given level x is studied. Asymptotic properties of the distribution are given for a large stress level x and for the tail of the distribution for fixed finite stress level x. Simulation results are given to assess the accuracy of using the asymptotic results to approximate the distribution.

1. INTRODUCTION

Many physical structures are subject to varying physical loads from various sources: wind, snow, and earthquakes being examples. In many instances the total load experienced by a structure varies in time in an apparently random fashion. Certain load components, e.g. those of snow and ice accumulation, vary rather slowly; others such as those associated with winds or earthquakes occur more nearly as impulses. The problem is to design structures to withstand a coincidence of such loads with approximately a prescribed (high) probability. In engineering terms we wish to work towards developing a rational safety factor criterion for designing structures to withstand the combination of loads anticipated.

In this paper we will study the time for the load combination process, $\{X(t);\ t \geq 0\}$ to exceed a given stress level for the case in which the load combination process is a regenerative process taking non-negative values (c.f. Cinlar [1975], page 298). Many of the models for load combinations that have been studied are regenerative processes; (cf. Pearce and Wen [1983], Wen [1981], Shanthikumar and Sumita [1983]).

EXAMPLE 1. The load combination process is the superposition of two load types: shock loads and constant loads. A constant load (e.g. caused by snow or rain accumulation) exists at one level for a random time and then changes to a new level. Let $Y(t)$ denote the constant load magnitude at time t. The instants of change in the magnitude of $Y(t)$ occur according to

a renewal process with inter-renewal distribution H; successive magnitudes of the constant load process are independent identically distributed with distribution F. Impulse or "shock" loads (e.g. caused by wind gusts or earthquakes) occur at random moments and have time duration of length 0. Let $Z(t)$ denote the shock load magnitude at time t. Given the constant load magnitude at time t, $Y(t) = y$, the probability a shock load will occur in the time interval $[t,t+h]$ is $\mu(y)h + o(h)$; the magnitude of the shock load is conditionally independent of everything given $Y(t) = y$ and has distribution $G(y;\cdot)$.

The load combination process magnitude at time t is $X(t) = Y(t) + Z(t)$. $\{X(t);\ t \geq 0\}$ is a regenerative process with regeneration times the times of change of the constant load process.

Gaver and Jacobs [1981] studied a special case of the above model in which the interrenewal distribution H is exponential, and the shock load process is a compound Poisson process independent of the constant load process with shock arrival rate μ and magnitude distribution function G. Other similar load combination models have been studied in the past; cf. Peir and Cornell [1973], Wen [1977], Pearce and Wen [1983].

Let $X(t)$ denote the magnitude of the load combination process at time t. The process $\{X(t),\ t \geq 0\}$ is assumed to be a regenerative process. Let

$$T_x = \inf\{t \geq 0 : X(t) \geq x\} , \tag{1.1}$$

the first time the load combination exceeds a level x.

Section 2 is concerned with asymptotic distribution of T_x as $x \to \infty$. It is shown that under certain assumptions the distribution of the normalized random variable $T_x(E[T_x]^{-1})$ is approximately unit exponential for large x, and error bounds on the rate of convergence are obtained. This result is related to that of Keilson [1979, page 134].

In Section 3 the tail of the distribution of T_x for finite x, $P\{T_x > t\}$, will be studied. An asymptotic result concerning the exponentiality of $P\{T_x > t\}$ for large t is given.

In Section 4 simulation results are presented to study the accuracy of using the two asymptotic results to approximate the distribution of T_x.

2. THE DISTRIBUTION OF THE FIRST PASSAGE TIME FOR THE LOAD COMBINATION PROCESS

Let $\{X(t);\ t \geq 0\}$ be a regenerative process taking non-negative values representing the load combination process; [cf. Cinlar [1975], page 298]. Let S_n denote the nth regeneration time; $\{S_n\}$ is a renewal process. We will assume there is a regeneration at time 0. Let T_x be as in (1.1) and put $\phi(\xi) = E[\exp(i\xi T_x)]$, the Fourier transform of T_x. A renewal theoretic argument yields

$$\phi(\xi) = E[\exp\{i\xi T_x\};T_x \leq S_1] + E[\exp\{i\xi T_x\};S_1 < T_x] \tag{2.1}$$

$$= E[\exp\{i\xi T_x\};T_x \leq S_1] + E[\exp\{i\xi S_1\};S_1 < T_x]\phi(\xi)$$

Thus,

$$\phi(\xi) = \frac{E[e^{i\xi T_x}; T_x < S_1]}{1 - E[e^{i\xi S_1}; S_1 < T_x]} . \tag{2.2}$$

Similarly, if $m(x) = E[T_x]$, then

$$\begin{aligned} m(x) &= E[T_x; T_x \leq S_1] + E[T_x; T_x > S_1] \\ &= E[T_x; T_x \leq S_1] + E[S_1; T_x > S_1] \\ &\quad + P\{T_x > S_1\} m(x) . \end{aligned} \tag{2.3}$$

Therefore,

$$m(x) = \frac{E[\min(T_x, S_1)]}{1 - P\{T_x > S_1\}} = \frac{E[\min(T_x, S_1)]}{P\{T_x \leq S_1\}} . \tag{2.4}$$

The following assumptions will be made for the remainder of this paper.

$$0 < E[S_1^2] < \infty \quad ; \tag{2.5}$$

$$\lim_{x \to \infty} P\{T_x \leq S_1\} = 0 . \tag{2.6}$$

It now follows from (2.4)-(2.6) that

$$\lim_{x \to \infty} m(x) = \infty . \tag{2.7}$$

LEMMA 1.

$$\frac{1}{|\xi|}\left|\phi\left(\frac{\xi}{m(x)}\right) - \frac{1}{1 - i\xi}\right| E[S_1;S_1 < T_x]$$

$$\leq \beta(x)\,\frac{E[S_1^2]}{2E[\min(S_1,T_x)]} + E[S_1^2]^{1/2}\beta(x)^{1/2}$$

$$+ |\xi|\beta(x)^{3/2}\,\frac{E[S_1^2]^{1/2}}{2E[\min(S_1,T_x)]}$$

where $\beta(x) = P\{T_x \leq S_1\}$.

Proof. It follows from (2.2) that there are random variables ε_1 and ε_2 such that

$$\phi\left(\frac{\xi}{m(x)}\right) = E[1 + \frac{i\xi}{m(x)} T_x + \varepsilon_1;T_x \leq S_1] \tag{2.8}$$

$$\times [1 - E[1 + \frac{i\xi}{m(x)} S_1 + \varepsilon_2;S_1 < T_x]]^{-1}$$

$$= [P\{T_x \leq S_1\} + \frac{i\xi}{m(x)} E[T_x;T_x \leq S_1] + E[\varepsilon_1;T_x \leq S_1]]$$

$$\times [P\{T_x \leq S_1\} - \frac{i\xi}{m(x)} E[S_1;S_1 < T_x] - E[\varepsilon_2;S_1 < T_x]]^{-1}$$

$$= (m(x)P\{T_x \leq S_1\} + i\xi E(T_x;T_x \leq S_1) + m(x)E[\varepsilon_1;T_x \leq S_1])$$

$$\times [m(x)P\{T_x \leq S_1\} - i\xi E[S_1;S_1 < T_x] - m(x)E[\varepsilon_2;S_1 > T_x]]^{-1}$$

$$\equiv (a + i\xi b) \times (c - i\xi d)^{-1}.$$

The difference between the Fourier transform of $T_x(E[T_x]^{-1})$ and that for a unit exponential is

$$[(a + i\xi b) \times (c - i\xi d)^{-1}] - (1 - i\xi)^{-1} \tag{2.9}$$

$$= [(a-c+\xi^2 b) + i\xi(b-a+d)] \times [(c-i\xi d)(1-i\xi)]^{-1}$$

$$\equiv \frac{NUM}{DEN} .$$

The term

$$a - c + \xi^2 b = m(x)P\{T_x \leq S_1\} + m(x)E[\varepsilon_1;T_x \leq S_1] \tag{2.10}$$

$$- m(x)P\{T_x \leq S_1\} + m(x)E(\varepsilon_2;S_1 < T_x]$$

$$+ \xi^2 E[T_x;T_x \leq S_1]$$

$$= m(x)([E[\varepsilon_1;T_x \leq S_1] + E[\varepsilon_2;S_1 < T_x])$$

$$+ \xi^2 E[T_x;T_x \leq S_1].$$

It follows from (2.4) that the term

$$b - a + d = E[T_x;T_x \leq S_1] - [m(x)P\{T_x \leq S_1\} + m(x)E[\varepsilon_1;T_x \leq S_1]]$$

$$+ E[S_1;S_1 < T_x] \tag{2.11}$$

$$b - a + d = E[\min(T_x,S_1)] - E[\min(T_x,S_1)] - m(x)E[\varepsilon_1;T_x \le S_1]$$

$$= -m(x)E[\varepsilon_1;T_x \le S_1].$$

Thus,

$$|NUM|^2 = \{m(x)(E[\varepsilon_1;T_x \le S_1] + E[\varepsilon_2;S_1 < T_x]) \tag{2.12}$$

$$+ \xi^2 E[T_x;T_x \le S_1]\}^2 + \xi^2(-m(x)E[\varepsilon_1;T_x \le S_1])^2.$$

From Lemma 1 on page 512 of Feller [1971]

$$E[|\varepsilon_1|;T_x \le S_1] \le \frac{1}{2}(\frac{\xi}{m(x)})^2 E[T_x^2;T_x \le S_1] \tag{2.13}$$

$$E[|\varepsilon_2|;S_1 < T_x] \le \frac{1}{2}(\frac{\xi}{m(x)})^2 E[S_1^2;S_1 < T_x] \ . \tag{2.14}$$

Thus

$$|NUM| \le \frac{1}{2}(\frac{\xi^2}{m(x)})(E[T_x^2;T_x \le S_1] + E[S_1^2;S_1 < T_x]) \tag{2.15}$$

$$+ \xi^2 E[T_x;T_x \le S_1] + |\xi| \ \frac{\xi^2}{m(x)} \ \frac{1}{2} \ E[T_x^2;T_x \le S_1].$$

Further,

$$|c - i\xi d| \ge |\xi| \ E[S_1;S_1 < T_x] \ . \tag{2.16}$$

Thus

$$\left|\frac{a + i\xi b}{c - i\xi d} - \frac{1}{1 - i\xi}\right| E[S_1; S_1 < T_x] \tag{2.17}$$

$$\le \frac{|\xi|}{m(x)} \frac{1}{2} (E[T_x^2; T_x \le S_1] + E[S_1^2; S_1 < T_x])$$

$$+ |\xi| E[T_x; T_x \le S_1] + \frac{\xi^2}{m(x)} \frac{1}{2} E[T_x; T_x \le S_1].$$

Application of Schwartz' Inequality yields

$$\frac{1}{|\xi|} \left|\phi\left(\frac{\xi}{m(x)}\right) - \frac{1}{1 - i\xi}\right| E[S_1; S_1 < T_x] \tag{2.18}$$

$$\le \frac{1}{2m(x)} E[S_1^2] + \left(1 + \frac{|\xi|}{2m(x)}\right) E[T_x; T_x \le S_1]$$

$$\le \frac{1}{2m(x)} E[S_1^2] + \left(1 + \frac{|\xi|}{2m(x)}\right) (E[S_1^2]^{1/2} P\{T_x \le S_1\}^{1/2}).$$

Let $\beta(x) = P\{T_x \le S_1\}$. Then

$$m(x) = \frac{E[\min(T_x, S_1)]}{\beta(x)} \quad . \tag{2.19}$$

Thus,

$$\frac{1}{|\xi|} \left|\phi\left(\frac{\xi}{m(x)}\right) - \frac{1}{1 - i\xi}\right| E[S_1; S_1 < T_x] \tag{2.20}$$

$$\le \beta(x) \frac{E[S_1^2]}{2E[\min(S_1, T_x)]} + E[S_1^2]^{1/2} \beta(x)^{1/2}$$

$$+ |\xi| \frac{\beta(x)^{3/2} E[S_1^2]^{1/2}}{2E[\min(S_1, T_x)]} \quad .$$

THEOREM 1. Under Assumptions (2.5)-(2.6)

$$|P\{\frac{T_x}{m(x)} \leq y\} - (1 - e^{-y})| \leq O(P\{T_x \leq S_1\}^{1/10}) .$$

Proof. It follows from Lemma 1 that

$$|\xi|^{-1} e^{i\xi}|E[\exp\{i\xi\frac{T_x}{m(x)}\}] - (1-i\xi)^{-1})|E[S_1;S_1 < T_x] \qquad (2.21)$$

$$\leq |\xi|(\beta(x) \frac{E[S_1^2]}{2E[\min(S_1,T_x)]} + E[S_1^2]^{1/2} \beta(x)^{1/2})$$

$$+ |\xi|^2 (\beta(x)^{3/2} \frac{E[S_1^2]^{1/2}}{2E[\min(S_1,T_x)]})$$

where $\beta(x) = P\{T_x \leq S_1\}$ as before.

Thus, applying Lemma 2 on page 538 of Feller [1971] for $T = \beta(x)^{-1/5}$,

$$\pi|P\{T_x \leq y\} - (1 - e^{-y})| \leq 24\beta(x)^{1/5}$$

$$+ E[S_1;S_1 < T_x]\{\beta(x)^{-2/5}[\beta(x) \frac{E[S_1^2]}{2E[\min(S_1,T_x)]} + E[S_1^2]^{1/2}\beta(x)^{1/2}]$$

$$+ \beta(x)^{-3/5} \frac{2}{3} \beta(x)^{3/2} \frac{E[S_1^2]^{1/2}}{2E[\min(S_1,T_x)]}\}.$$

The result now follows.

EXAMPLE. In Example 1

$$\beta(x) = \overline{F}(x) + \int_0^x F(dy) \int_0^\infty H(dt)[1 - \exp\{-\mu(y)\overline{G}(y,x-y)t\}]$$

$$= 1 - \int_0^x F(dy)\hat{h}(\mu(y)\overline{G}(y,x-y)) \qquad (2.22)$$

where $\hat{h}(s) = \int_0^\infty e^{-st}H(dt)$ is the Laplace transform of H and $\overline{F}(t) = 1 - F(t)$. If C and S are two independent random variables having distributions F and H respectively, then

$$\beta(x) = 1 - E[\exp\{-\mu(C)\overline{G}(C,x-C)S\}; C \leq x] \ . \qquad (2.23)$$

Consider two independent load combination processes $\{X_1(t); t \geq 0\}$ and $\{X_2(t); t \geq 0\}$ of the type in Example 1. Assume that the conditional distribution of the shock load process, given the constant load process, is the same for both load combinati processes; the constant load magnitudes have distributions F_1 and F_2 respectively with $\overline{F}_1(t) \leq \overline{F}_2(t)$; and the times between constant load changes have distributions H_1 and H_2 respectively with $\overline{H}_1(t) \leq \overline{H}_2(t)$. Let C_i and S_i have distributions F_i and H_i respectively for $i = 1,2$. If $y \to \mu(y)\overline{G}(y,x-y)$ is an increasing function of y for $y \leq x$, then

$$\beta_1(x) \equiv 1 - E[e^{-\mu(C_1)\overline{G}(C_1,x-C_1)S_1}; C_1 \leq x]$$

$$\beta_1(x) \leq 1 - E[e^{-\mu(C_2)\overline{G}(C_2,x-C_2)S_2};C_2 \leq x]$$

$$\equiv \beta_2(x) \ . \tag{2.24}$$

Thus, Theorem 1 suggests that the convergence to exponential of the distribution of the time for the load combination process to exceed a level x is faster for load combination process 1 than that for process 2. This behavior has been seen in the simulation studies reported in Section 4.

3. THE TAIL OF THE DISTRIBUTION OF T_x FOR FINITE x

In this section we will study the behavior of $P\{T_x > t\}$ for finite x and large t. The probability, $P\{T_x > t\}$, satisfies the following renewal equation

$$P\{T_x > t\} = P\{S_1 > t, T_x > t\} + \int_0^t L_x(du)P\{T_x > t-u\} \tag{3.1}$$

where

$$L_x(t) = P\{S_1 \leq t, T_x > S_1\} \ . \tag{3.2}$$

We will assume $L_x(0) = 0$.

Following the argument on page 376 of Feller [1971] we will assume that for each x there exists a constant $\kappa(x)$ such that

$$\int_0^\infty e^{\kappa(x)u} L_x(du) = 1 \ , \tag{3.3}$$

$$\gamma(x) = \int_0^\infty te^{\kappa(x)t} L_x(dt) \tag{3.4}$$

is finite, and the function

$$g_x(t) = e^{\kappa(x)t} P\{S_1 > t, T_x > t\} \tag{3.5}$$

is directly Riemann integrable.

It now follows from the Key Renewal Theorem that

$$\lim_{t\to\infty} e^{\kappa(x)t} P\{T_x > t\} = \frac{1}{\gamma(x)} \int_0^\infty g_x(t)dt \ . \tag{3.6}$$

Since g is assumed to be directly Riemann integrable, integrating (3.3) by parts yields

$$1 = P\{T_x > S_1\} + \kappa(x) \int_0^\infty e^{\kappa(x)t} P\{S_1 > t, T_x > S_1\}dt. \tag{3.7}$$

Thus

$$\kappa(x) = \frac{P\{T_x \leq S_1\}}{\int_0^\infty e^{\kappa(x)t} P\{S_1 > t, T_x > S_1\}dt} \ . \tag{3.8}$$

It follows from (2.4) that

$$\kappa(x)m(x) = \frac{E[\min(T_x,S_1)]}{\int_0^\infty e^{\kappa(x)t}\, P\{S_1 > t, T_x > S_1\}dt} . \qquad (3.9)$$

The defining equation for $\kappa(x)$, (3.3), (3.8), and (2.6) imply that as a function of x, $\kappa(x)$ is nonnegative and decreasing with $\lim_{x\to\infty} \kappa(x) = 0$. If it is further assumed that there exists $\theta > 0$ such that $E[e^{\theta S_1}] < \infty$, then it follows from the dominated convergence theorem and (3.9) that

$$\lim_{x\to\infty} \kappa(x)m(x) \equiv 1 . \qquad (3.10)$$

EXAMPLE. In Example 1, assume that $\overline{H}(t) = e^{-\lambda t}$, $\mu(y) \equiv \mu$, and $G(y,x) \equiv G(x)$. Then $\kappa(x)$ satisfies the equation

$$1 = \int_0^x F(dy) \frac{\lambda}{\lambda + \mu\overline{G}(x-y) - \kappa(x)} \qquad (3.11)$$

where $\overline{G}(x) = 1 - G(x)$.

Further,

$$g_x(t) = \int_0^x e^{\kappa(x)t}\, e^{-\ t}\, e^{-\ \overline{G}(x-y)t}\, F(dy) .$$

Thus

$$\int_0^\infty g_x(t)\,dt = \int_0^x F(dy)\,\frac{1}{\lambda + \mu\bar{G}(x-y) - \kappa(x)} = \frac{1}{\lambda}\,. \tag{3.12}$$

Therefore,

$$\lim_{t\to\infty} e^{\kappa(x)t}\, P\{T_x > t\} = \frac{1}{\lambda\gamma(x)} \tag{3.13}$$

where

$$\gamma(x) = \int_0^\infty \int_0^x F(dy)\; t\, e^{\kappa(x)} \lambda e^{-\lambda t}\, e^{-\mu\bar{G}(x-y)t}\, dt$$

$$= \int_0^x F(dy)\,\frac{\lambda}{(\lambda + \mu\bar{G}(x-y) - \kappa(x))^2}\,. \tag{3.14}$$

If it is further assumed that $\lambda = \mu = 1$, then it follows from (3.9) and (3.11) that

$$\kappa(x)m(x) = \frac{\int_0^x F(dy)\,\frac{1}{1+\bar{G}(x-y)}}{\int_0^x F(dy)\,\frac{1}{1+\bar{G}(x-y)-\kappa(x)}} \tag{3.15}$$

$$\leq \int_0^x F(dy)\,\frac{1}{1+\bar{G}(x-y)}$$

Thus $\kappa(x) \leq \frac{1}{m(x)}$ in this case.

In the further special case in which $\overline{F}(x) = \overline{G}(x) = e^{-x}$, $\lambda = 1$ and $\mu = 1$, $\kappa(x)$ satisfies the equation

$$1 = f(\kappa(x)) \equiv \int_0^x e^{-y} \frac{1}{1 + e^{-(x-y)} - \kappa(x)} dy \qquad (3.15)$$

where

$$f(1) = \frac{1}{2}[e^{x} - e^{-x}] \qquad (3.16)$$

and for $y \neq 1$ such that $0 \leq y \leq 1 - e^{-x}$,

$$f(y)(1-y)^2 = (1-y)(1-e^{-x}) - e^{-x}[\ln((1-y)+e^{-x}) - \ln((1-y)e^{-x}+e^{-x})] \ . \qquad (3.17)$$

Further,

$$\gamma(x) = \int_0^x \frac{1}{(1 + e^{-(x-y)} - \kappa(x))^2} e^{-y} dy; \quad \text{or}$$

$$\gamma(x)(1-\kappa(x))^3 = (1-\kappa(x))(1-e^{-x}) - 2e^{-x}[\ln((1-\kappa(x))+e^{-x})$$

$$- \ln((1-\kappa(x))e^{-x}+e^{-x})] \ . \qquad (3.18)$$

4. A SIMULATION STUDY OF THE ACCURACY OF THE EXPONENTIAL APPROXIMATIONS TO THE FIRST PASSAGE TIME DISTRIBUTION

In this section some results of a simulation study of the accuracy of using the asymptotic results of Sections 2 and 3 to approximate the distribution of T_x are reported. All simulations were carried out on an IBM 3033 computer at the Naval Postgraduate School using the LLRANDOM II random number generating package (see Lewis and Uribe (1981)).

The model of Example 1 was simulated for various cases of distributions H, F, G and shock arrival rate μ. Each realization simulated the sample path of the process $\{X(t); t \geq 0\}$ and the first passage times $0 \leq T_{x_1} \leq T_{x_2} \leq \cdots \leq T_{x_n}$ were recorded for several levels $x_1 \leq x_2 \leq \cdots \leq x_n$. The number of replications was 5,000. Sample moments and quantiles were computed for each T_{x_i}. A more detailed account of the simulation can be found in Noh (1984).

4.1 The Exponential Approximation $P\{T_x > t\} = \exp\{-E[T_x]t\}$

a. Model A

Tables 1-1, 1-2 and 1-3 report the simulation results for the model in which H = F = G are all unit exponential and $\mu \equiv 1$. Table 1-1 reports simulated sample mean and coefficient of variation for T_x for various levels of x. As expected, the simulated coefficient of variation approaches the exponential distribution's value of 1 as x gets larger. To assess the quality of the exponential approximation $1 - \exp\{E[T_x]^{-1}t\}$ to the distribution of T_x, quantiles from the simulated data were computed and compared to the approximating exponential

TABLE 1-1

Simulated Moments for T_x in the Case F = G = H = exp(1) and μ = 1

x-Level	$\hat{E}[T_x]$	Coeff Var$[T_x]$
0.50	0.276 *(0.01)	1.984
1.00	0.669 (0.014)	1.451
2.00	2.000 (0.033)	1.177
3.00	4.878 (0.074)	1.080
4.00	11.539 (0.167)	1.024
5.00	26.945 (0.382)	1.002

*( ) is the standard error of the mean

quantile q_α

$$q_\alpha = -E[T_x] \ln(1-\alpha) \tag{4.1}$$

where for this model $E[T_x]$ can be computed analytically and is found in Gaver and Jacobs [1981]. Table 1-2 reports the simulated quantiles with the approximating exponential quantiles below in parentheses. As expected, the exponential approximation is better for the large level $x = 5$ than for $x = 0.5$. The approximation is also better for $\alpha > 0.5$.

One way the distribution of T_x differs from an exponential is that it has an atom at 0; in particular for the model A $P\{T_x = 0\} = 1 - F(x) = e^{-x}$. The sample quantiles for the simulated conditional distribution of T_x given $T_x > 0$ appear in Table 1-3. Below the simulated quantiles in parentheses appear approximating exponential ones computed as

$$q_\alpha^c = \frac{-E[T_x]}{P\{T_x > 0\}} \ln(1-\alpha) \ . \tag{4.2}$$

The quantiles of the simulated conditional distribution are much closer to their exponential approximation than those for the unconditional distribution.

b. Model B

Tables 2-1, 2-2, and 2-3 report simulation results for the model which is the same as Model A except that the distribution of the constant load magnitudes F is exponential with mean 1/2. The simulated means and coefficient of variations

TABLE 1-2

Simulated α-Quantiles for the Distribution of T_x for the Model with $F = G = H =$ Unit Exponential and $\mu = 1$

α:	0.1	0.2	0.25	0.30	0.50	0.75	0.80	0.90	0.95	0.98	0.99
Level x											
0.5	0.0	0.0	0.0	0.0	0.0	0.335	0.496	0.959	1.432	2.007	2.485
	*(0.03)	(0.64)	(0.083)	(0.102)	(0.199)	(0.398)	(0.462)	(0.661)	(0.860)	(1.123)	(1.322)
1.00	0.0	0.0	0.0	0.0	0.261	0.972	1.200	1.918	2.683	3.631	4.358
	(0.071)	(0.151)	(0.195)	(0.242)	(0.470)	(0.940)	(1.091)	(1.561)	(2.031)	(2.652)	(3.122)
2.00	0.0	0.171	0.313	0.454	1.233	2.877	3.393	5.111	6.490	8.875	10.419
	(0.215)	(0.454)	(0.586)	(0.726)	(1.411)	(2.823)	(3.277)	(4.688)	(6.099)	(7.965)	(9.376)
3.00	0.252	0.847	1.164	1.525	3.175	6.916	8.099	11.497	15.012	20.501	24.655
	(0.525)	(1.113)	(1.434)	(1.778)	(3.456)	(6.912)	(8.025)	(11.481)	(14.937)	(19.505)	(22.961)
4.00	0.990	2.229	2.944	3.726	7.962	16.339	18.902	26.659	34.861	47.158	54.212
	(1.225)	(2.594)	(3.344)	(4.146)	(8.057)	(16.114)	(18.708)	(26.765)	(34.822)	(45.473)	(53.530)
5.00	2.404	5.738	7.584	9.387	18.598	37.717	44.380	62.979	81.667	105.922	123.817
	(2.837)	(6.009)	(7.748)	(9.606)	(18.66)	(37.334)	(43.344)	(62.011)	(80.678)	(105.355)	(124.022)

*( ) Approximating exponential quantile

TABLE 1-3

Simulated α-Quantiles for the Conditional Distribution of T_x, Given $T_x > 0$, for the Model with F = G = H = Unit Exponential and $\mu = 1$

α:	0.1	0.2	0.25	0.3	0.5	0.75	0.80	0.90	0.95	0.98	0.99
Level x											
0.5	0.075 *(0.074)	0.159 (0.156)	0.207 (0.201)	0.264 (0.250)	0.505 (0.485)	0.966 (0.971)	1.120 (1.127)	1.590 (1.612)	2.007 (2.098)	2.621 (2.739)	3.072 (3.225)
1.0	0.112 (0.110)	0.238 (0.232)	0.295 (0.300)	0.373 (0.272)	0.727 (0.722)	1.445 (1.444)	1.667 (1.677)	2.384 (2.399)	3.117 (3.121)	4.295 (4.075)	4.859 (4.797)
2.0	0.231 (0.244)	0.481 (0.516)	0.628 (0.665)	0.796 (0.825)	1.575 (1.603)	3.203 (3.206)	3.705 (3.722)	5.353 (5.325)	6.927 (6.927)	9.239 (9.046)	10.737 (10.649)
3.0	0.510 (0.541)	1.094 (1.146)	1.442 (1.478)	1.783 (1.832)	3.435 (3.560)	7.218 (7.121)	8.306 (8.267)	11.763 (11.827)	15.419 (15.388)	20.761 (20.094)	24.913 (23.654)
4.0	1.187 (1.239)	2.404 (2.623)	3.172 (3.382)	3.957 (4.193)	8.126 (8.148)	16.575 (16.297)	19.076 (18.920)	26.773 (27.068)	35.131 (35.216)	47.471 (45.988)	54.212 (54.136)
5.0	2.523 (2.857)	5.877 (6.051)	7.775 (7.801)	9.552 (9.672)	18.691 (18.792)	37.877 (37.594)	44.527 (43.645)	63.174 (62.442)	81.777 (81.239)	105.922 (106.087)	123.817 (124.884)

*( ) the approximating exponential quantile

TABLE 2-1

Simulated Moments for Model B

Level x	$E[\hat{T}_x]$	Coeff $\hat{Var}[T_x]$
0.5	0.577 *(0.010)	1.453
1.0	1.329 (0.022)	1.144
2.0	3.980 (0.058)	1.036
3.0	10.530 (0.150)	1.006
4.0	28.014 (0.395)	0.997
5.0	75.425 (1.070)	1.003

*( ) is the standard error of the sample mean

TABLE 2-2

Simulated α-Quantiles for the Distribution of T_x for Model B

α:	0.1	0.2	0.25	0.3	0.5	0.75	0.80	0.90	0.95	0.98	0.99
Level x											
0.5	0.0 *(0.60)	0.0 (0.126)	0.0 (0.163)	0.0 (0.202)	0.232 (0.392)	0.843 (0.785)	1.048 (0.911)	1.660 (1.304)	2.281 (1.696)	3.082 (2.215)	3.856 (2.607)
1.0	0.0 (0.138)	0.118 (0.293)	0.219 (0.377)	0.327 (0.468)	0.838 (0.909)	1.913 (1.818)	2.267 (2.111)	3.351 (3.020)	4.309 (3.930)	5.913 (5.132)	6.693 (6.041)
2.0	0.329 (0.422)	0.822 (0.894)	1.084 (1.152)	0.329 (1.428)	2.699 (2.775)	5.491 (5.551)	6.441 (6.444)	9.237 (9.220)	12.194 (11.995)	16.075 (15.664)	19.031 (18.400)
3.0	1.072 (1.128)	2.321 (2.389)	2.954 (3.080	3.643 (3.819)	7.180 (7.422)	14.649 (14.843)	17.429 (17.232)	24.906 (24.654)	31.688 (32.075)	41.991 (41.886)	48.155 (49.308)
4.0	2.907 (2.988)	6.253 (6.327)	8.247 (8.157)	10.036 (10.114)	19.494 (19.655)	38.714 (39.309)	45.090 (45.637)	65.697 (65.291)	84.429 (84.946)	107.641 (110.928)	123.499 (130.582)
5.0	8.137 (7.975)	17.063 (16.890)	21.753 (21.776)	26.729 (26.998)	52.403 (52.466)	104.272 (104.933)	120.132 (121.823)	170.902 (174.290)	224.441 (226.756)	298.617 (296.113)	350.441 (348.579)

*( ) is the approximating exponential quantile

TABLE 2-3

Simulated α-Quantiles for the Conditional Distribution of T_x, Given $T_x > 0$ for Model B

α:	0.10	0.20	0.25	0.30	0.5	0.75	0.80	0.90	0.95	0.98	0.99
Level x											
0.5	0.098 *(0.095)	0.212 (0.202)	0.275 (0.261)	0.336 (0.323)	0.629 (0.628)	1.255 (1.256)	1.436 (1.458)	2.019 (2.085)	2.711 (2.713)	3.546 (3.543)	4.232 (4.171)
1.0	0.157 (0.162)	0.349 (0.344)	0.443 (0.443)	0.543 (0.549)	1.063 (1.067)	2.157 (2.134)	2.488 (2.478)	3.580 (3.545)	4.533 (4.612)	6.065 (6.023)	6.841 (7.090)
2.0	0.399 (0.428)	0.899 (0.905)	1.146 (1.167)	1.408 (1.447)	2.772 (2.812)	5.566 (5.625)	6.502 (6.530)	9.277 (9.343)	12.202 (12.155)	16.086 (15.873)	19.031 (18.686)
3.0	1.098 (1.113)	2.348 (2.357)	2.978 (3.038)	3.667 (3.767)	7.220 (7.321)	14.689 (14.641)	17.450 (16.998)	24.933 (24.319)	31.743 (31.640)	41.991 (41.317)	48.155 (48.638)
4.0	2.907 (2.952)	6.253 (6.252)	8.247 (8.061)	10.036 (9.994)	19.494 (19.422)	38.714 (38.844)	45.090 (45.096)	65.697 (64.518)	84.429 (83.940)	107.641 (109.614)	123.499 (124.036)
5.0	8.137 (7.948)	17.063 (16.834)	21.753 (21.703)	26.729 (26.908)	52.403 (52.291)	104.272 (104.582)	120.132 (121.416)	170.902 (173.707)	224.441 (225.998)	298.617 (295.122)	350.441 (347.413)

*Approximating exponential quantiles

appear in Table 2-1. An analytical expression for $E[T_x]$ for this model appears in Gaver and Jacobs [1981]. This expression is used in the approximating quantiles (4.1) and (4.2). The approximating quantiles for the conditional distribution of T_x given $T_x > 0$ which appear in Table 2-3 are closer to their corresponding simulated quantiles than the approximating quantiles for the unconditional distribution. Comparison of Tables 2-2 and 2-1 for the $x \geq 2$ and $\alpha \leq 0.3$ suggests that the distribution of T_x is converging faster to an exponential for Model B than for Model A. It follows from (2.2) that $P\{T_x < S_1\}$ is smaller for Model B than for Model A. Thus, Theorem 1 suggests that the convergence of the distribution of T_x to exponential should be faster for Model B than for Model A.

4.2 The Exponential Approximation $\frac{1}{\gamma(x)} e^{-\kappa(x)t}$

In this subsection simulation will be used to study the exponential approximation suggested by the asymptotic result (3.6). This is an approximation for $P\{T_x > t\}$ for fixed finite x; it should be more accurate for t large.

Two cases of Model 1 were simulated. In both cases, shock loads arrive according to a Poisson process with rate 1 and constant loads change magnitude at the times of arrival of a Poisson process with rate 1; the shock level magnitudes have an exponential distribution with mean 1. In Case A, the distribution of the constant load magnitude is exponential with mean 1; in Case B, it is exponential with mean 1/2.

In both cases considered, it is possible to determine analytical expressions for the integrals determining $\kappa(x)$ and

$\gamma(x)$. The value for $\kappa(x)$ was found numerically. Values of $\kappa(x)$ and $\gamma(x)$ for the two models can be found in Tables (3-1) and (4-1). Note that as x increases $\kappa(x)$ decreases and approaches $E[T_x]^{-1}$. As x increases $\gamma(x)$ decreases and approaches 1. As expected $\kappa(x) \leq E[T_x]^{-1}$ for all levels of x.

To assess the accuracy of the exponential approximation, quantiles of the simulated data were computed. These quantiles appear in Tables (3-2) and (4-2). For each level x, the first row gives the simulated quantile, the second row gives the approximating exponential quantile

$$Q_\alpha^* = -\frac{1}{\kappa(x)} \ln(\gamma(x)(1-\alpha)) \; ; \qquad (4.3)$$

and the third row gives the approximating exponential quantile

$$Q_\alpha^+ = -E[T_x] \ln(1-\alpha). \qquad (4.4)$$

The exponential approximation (4.3) is in general closer to the simulated quantile than (4.4). However the two approximations become closer as x gets larger. As expected (4.3) approximates well the simulated quantile Q_α for $\alpha > 0.75$ for all values of x. However if x is sufficiently large (4.3) approximates fairly well the simulated Q_α for α as small as 0.1. A comparison of Tables (3-2) and (4-2) suggests once again that the convergence of the distribution of T_x to exponential is faster for Model B than for Model A.

TABLE 3-1

Values of $\kappa(x)$ and $\gamma(x)$ for Model A

Level x	$\kappa(x)$	$E[T_x]^{-1}$	$\gamma(x)$
0.2	1.708	9.492	5.942
0.4	1.459	4.486	3.254
0.6	1.247	2.814	2.364
0.8	1.066	1.977	1.923
1.0	0.912	1.475	1.670
1.2	0.780	1.142	1.502
1.4	0.667	0.906	1.386
1.6	0.570	0.730	1.303
1.8	0.487	0.596	1.238
2.0	0.417	0.491	1.193
2.2	0.355	0.407	1.155
2.4	0.303	0.339	1.125
2.6	0.259	0.284	1.102
2.8	0.221	0.238	1.085
3.0	0.188	0.201	1.068

TABLE 4-1

Values of $\kappa(x)$ and $\gamma(x)$ for Model B

Level x	$\kappa(x)$	$E[T_x]^{-1}$	$\gamma(x)$
0.2	1.563	4.760	3.109
0.4	1.237	2.265	1.860
0.6	0.975	1.433	1.400
0.8	0.801	1.015	1.286
1.0	0.650	0.762	1.183
1.2	0.531	0.592	1.122
1.4	0.435	0.470	1.084
1.6	0.358	0.378	1.059
1.8	0.296	0.306	1.044
2.0	0.244	0.250	1.032
2.2	0.199	0.204	1.019
2.4	0.166	0.168	1.017
2.6	0.136	0.138	1.011
2.8	0.112	0.113	1.007
3.0	0.092	0.093	1.005

TABLE 3-2

Simulated Quantiles for Model A

α:	0.10	0.20	0.25	0.40	0.50	0.60	0.75	0.80	0.90	0.95	0.98	0.99
Level x												
0.20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.336	0.722	1.234	1.639
	*0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.305	0.711	1.247	1.653
	+0.11	0.024	0.030	0.054	0.073	0.097	0.146	0.170	0.243	0.316	0.412	0.485
0.60	0.0	0.0	0.0	0.0	0.0	0.095	0.449	0.619	1.151	1.744	2.484	3.086
	0.0	0.0	0.0	0.0	0.0	0.045	0.422	0.601	1.157	1.712	2.447	3.003
	0.037	0.079	0.102	0.182	0.246	0.326	0.493	0.572	0.818	1.065	1.390	1.637
1.00	0.0	0.0	0.0	0.065	0.249	0.467	0.971	1.208	1.999	2.779	3.689	4.506
	0.0	0.0	0.0	0.0	0.198	0.443	0.958	1.203	1.963	2.724	3.729	4.489
	0.071	0.151	0.195	0.346	0.470	0.621	0.940	1.091	1.561	2.031	2.652	3.122
1.60	0.0	0.007	0.115	0.455	0.774	1.151	1.984	2.392	3.558	4.766	6.393	7.623
	0.0	0.0	0.041	0.433	0.752	1.144	1.969	2.360	3.576	4.792	6.400	7.616
	0.144	0.305	0.394	0.699	0.949	1.254	1.898	2.203	3.152	4.101	5.356	6.305
2.00	0.0	0.198	0.332	0.845	1.267	1.839	2.941	3.462	5.114	6.813	9.104	10.895
	0.0	0.113	0.268	0.803	1.241	1.777	2.905	3.441	5.105	6.769	8.969	10.633
	0.215	0.454	0.536	1.040	1.411	1.866	2.823	3.277	4.688	6.099	7.965	9.376
2.60	0.117	0.523	0.797	1.662	2.347	3.191	4.913	5.814	8.513	11.420	15.234	17.831
	0.033	0.488	0.737	1.599	2.304	3.165	4.981	5.843	8.520	11.198	14.737	17.415
	0.371	0.786	1.013	1.798	2.440	3.226	4.880	5.666	8.106	10.546	13.772	16.212
3.00	0.272	0.872	1.184	2.380	3.337	4.458	6.892	8.177	12.086	15.933	20.633	23.946
	0.208	0.833	1.175	2.360	3.327	4.511	7.005	8.189	11.867	15.545	20.407	24.085
	0.325	1.113	1.434	2.547	3.456	4.569	6.912	8.025	11.481	14.937	19.505	22.961

* Quantile for exponential approximation (4.3)

\+ Quantile for exponential approximation (4.4)

TABLE 4-2

Simulated Quantiles for Model B

α:	0.10	0.20	0.25	0.40	0.50	0.60	0.75	0.80	0.90	0.95	0.98	0.99
Level x												
0.20	0.0	0.0	0.0	0.0	0.0	0.0	0.192	0.327	0.765	1.205	1.765	2.144
	*0.0	0.0	0.0	0.0	0.0	0.0	0.161	0.304	0.747	1.191	1.777	2.221
	+0.022	0.047	0.060	0.107	0.146	0.193	0.291	0.338	0.484	0.624	0.822	0.967
0.60	0.0	0.0	0.0	0.154	0.341	0.561	1.032	1.253	1.901	2.561	3.514	4.195
	0.0	0.0	0.0	0.179	0.366	0.595	1.077	1.306	2.017	2.728	3.668	4.379
	0.074	0.156	0.201	0.356	0.484	0.639	0.967	1.123	1.607	2.090	2.730	3.213
1.00	0.0	0.118	0.219	0.545	0.823	1.160	1.871	2.208	3.242	4.381	5.787	6.947
	0.0	0.085	0.184	0.527	0.808	1.151	1.874	2.217	3.284	4.350	5.759	6.826
	0.138	0.293	0.377	0.670	0.909	1.202	1.818	2.111	3.020	3.930	5.131	6.041
1.60	0.165	0.486	0.660	1.296	1.763	2.386	3.678	4.297	6.197	8.255	10.953	12.856
	0.136	0.465	0.645	1.269	1.779	2.403	3.717	4.340	6.278	8.216	10.778	12.716
	0.279	0.591	0.762	1.353	1.836	2.427	3.671	4.262	6.098	7.933	10.360	12.195
2.00	0.342	0.817	1.099	1.987	2.760	3.683	5.593	6.550	9.376	12.197	16.032	18.469
	0.305	0.788	1.053	1.969	2.717	3.632	5.561	6.477	9.321	12.165	15.925	18.769
	0.422	0.894	1.152	2.045	2.775	3.669	5.551	6.444	9.220	11.995	15.664	18.440
2.60	0.729	1.584	2.048	3.672	5.059	6.687	10.121	11.658	16.645	21.655	28.242	33.846
	0.694	1.558	2.032	3.669	5.007	6.645	10.095	11.732	16.819	21.906	28.631	33.718
	0.764	1.618	2.086	3.704	5.026	6.644	10.052	11.670	16.695	21.721	28.365	33.391
3.00	1.121	2.340	3.036	5.460	7.539	9.830	14.826	17.378	24.827	32.112	43.109	51.064
	1.093	2.371	3.071	5.492	7.470	9.891	14.989	17.410	24.930	32.449	42.390	49.910
	1.128	2.389	3.080	5.469	7.422	9.811	14.843	17.232	24.654	32.075	41.886	49.307

* Quantile for exponential approximation (4.3)

+ Quantile for exponential approximation (4.4)

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